POSSIBLE EXTENSION OF THE CHIRAL

PERTURBATION THEORY PROGRAM

J. SCHECHTER

Department of Physics, Syracuse University,
Syracuse, NY 13244-1130, USA

ABSTRACT

After a brief discussion of how chiral dynamics has evolved from the "universal V-A theory of weak interactions", we present some evidence that symmetry breaking for the vector meson multiplet is not simpler than but rather analogous to that for the pseudoscalar multiplet. This provides a motivation for speculating on how to extend in a systematic way the chiral perturbation theory program to include vectors.

1. Introduction

I would like to dedicate this paper to the memory of Professor Robert Marshak. As an ex-graduate student at Rochester I am grateful to him for establishing an intellectually stimulating and supportive Particle Physics group there. His enthusiasm for research and down-to-earth attitude were much appreciated by all of us.

Out of a career filled with many achievements in physics, Professor Marshak's chief one was the deduction, together with E.C.G. Sudarshan, of the "V-A" form of the weak interaction.[1] On the one hand, this theory provided a basis for understanding a wealth of experimental data. Equally important, it indicated that the relevant degrees of freedom of the observed "material" particles in nature were not the Dirac spinor fields but rather their left and right chiral projections. The left projections appeared in the weak interactions while both left and right were needed for the strong and electromagnetic interactions.

The most evident application of this idea to the strong interactions of the low lying - i.e. pseudoscalar meson - states requires us to increase the size of the "flavor" symmetry multiplet. Instead of treating the 0^- mesons as belonging to a 3×3 matrix Φ which transforms as $\Phi \to U\Phi U^{-1}$, $U^\dagger U = 1$ under the approximate symmetry group of the light quarks, we associate a 3×3 matrix S of scalar mesons with Φ to form an object $M = S + i\Phi$ which transforms as $M \to U_L M U_R^\dagger$ under separate left projected and right projected unitary groups.

Some consequences of this "linear" chiral symmetry approach were discussed by Prof. Marshak and collaborators in Ref. 2. However it turns out that another important physical ingredient is required. The vacuum of the strong interaction theory is not (even in the limit of massless light quarks) invariant under the separate U_L and U_R transformations, although the Lagrangian is invariant. As Nambu[3]

explained to the world, this implies that the pseudoscalar and scalar masses are drastically split from each other, the pseudoscalar masses being forced to become zero in the massless quark limit. This situation can be neatly handled by making the polar decomposition $M = HU$, $H = H^\dagger, U^\dagger = U^{-1}$ and " freezing out" the scalar field part H by setting $H = (F_\pi/2)\mathbb{1}$. The freezing out corresponds to sending the scalar masses to infinity while $F_\pi \simeq 0.132$ GeV is the pion decay constant. The name of this basic constant of strong interaction physics betrays its origin in the weak interactions. Note that $U \to U_L U U_R^\dagger$ under chiral transformations. U is a function of the ordinary pseudoscalar field matrix ϕ (which is simply related to Φ and S by a "point transformation") and one may conveniently set[4] $U = \exp(2i\phi/F_\pi)$. ϕ evidently behaves non-linearly under chiral transformations. Having isolated the proper degrees of freedom it remains only to note that the simplest invariant Lagrangian density formed with U,

$$\mathcal{L} = -\frac{F_\pi^2}{8} \operatorname{Tr} (\partial_\mu U \partial_\mu U^\dagger) + ... , \tag{1.1}$$

provides an accurate representation of QCD at very low energies (where perturbation theory fails). Indeed, many of the "current algebra" theorems, carefully discussed in the treatise "Theory of Weak Interactions in Particle Physics"[5] can be obtained using (1.1) in a simple way.

It seems remarkable that, although many years have passed since the introduction of the "V-A" theory, chiral symmetry is still an extremely active field of research. In the following, I shall very briefly describe some of the progress in the field and shall present some speculations related to further improvement of the approach.

2. Going Beyond Very Low Energies

Modifying the simple model (1.1) in an attempt to describe QCD over the full traditional low energy range (say up to about 1 GeV) has been the task of a generation. And it has still not been definitively accomplished. It is, of course, necessary to recognize that the chiral symmetry is broken by the light quark mass terms[6] in the fundamental QCD Lagrangian,

$$\mathcal{L}_{\mathrm{mass}} = -\hat{m}\bar{q}\mathcal{M}q, \tag{2.1}$$

where q is the column vector of up, down and strange quark fields, $\hat{m} = (m_u + m_d)/2$ and $\mathcal{M}$ is a dimensionless, diagonal matrix which can be expanded as follows:

$$\mathcal{M} = y\lambda_3 + T + xS, \tag{2.2}$$

with $\lambda_3 = \operatorname{diag}(1, -1, 0), \mathrm{T} = \operatorname{diag}(1, 1, 0)$ and $S = \operatorname{diag}(0, 0, 1)$. x and y are the quark mass ratios:

$$x = \frac{m_s}{\hat{m}}, \; y = -\frac{1}{2}\left(\frac{m_d - m_u}{\hat{m}}\right). \tag{2.3}$$

The minimal term at the effective Lagrangian level which can mock up (2.1) is:

$$\mathcal{L}_{SB} = \delta' \text{ Tr } [\mathcal{M}(U + U^\dagger - 2)], \tag{2.4}$$

where δ' is a numerical constant. Fitting (1.1) + (2.4) to the experimental pseudoscalar mass spectrum yields the standard determination of the quark mass ratios x and y.

Now, there are two (in principle) straightforward ways in which one might extend up in energy the description of low energy physics obtained by using (1.1) + (2.4) at tree level:

i. Including loop diagrams[7]

First compute the one loop corrections using (1.1) + (2.4) and keep terms quartic in the momenta. To eliminate the divergences add chiral invariant "counter-terms" quartic in derivatives. For this purpose count each power of $\mathcal{M}$ as two derivatives (The terms involving $\mathcal{M}$ will be chiral covariant rather than invariant). This scheme can be continued to higher orders in momenta. However since the starting Lagrangian is non-renormalizable there will be an infinite number of counterterms. This is not a worry in practice as we only expect to use the method up to a low finite order.

ii. Including additional physical particles

Even without considering any detailed models it is obvious that if we want to have an effective theory valid up to some energy we should include the physical particles whose masses lie in that energy range. How else would we get the right poles in the S-matrix at tree level? This point of view is buttressed by the Veneziano model[8] which shows how to get good high energy behavior by adding pole contributions from a realistic looking (infinite) set of particles. It also is supported by the leading large N_c approximation[9] to QCD in which one should keep only the tree diagrams involving all the physical mesonic states. The baryonic states can be obtained from this effective meson Lagrangian as Skyrme solitons.[10]

Which of these two approaches is superior, or is that even the right question to ask? The first approach, which has been systematized by Gasser and Leutwyler[7] is known as "chiral perturbation theory" (CPT). In practice it essentially amounts to making a complete list of chiral invariant and covariant counterterms, each with an unknown coupling constant. There are about ten of these at the quartic derivative order. With a convenient choice of the renormalization point, the contributions from the loop diagrams themselves (the "chiral logs") are typically negligible. It seems that the CPT approach is both solid and useful for improving the description of (1.1) + (2.4) at very low energies, say up to about 500 MeV in $\pi - \pi$ scattering. But going beyond this region forces us to face the enormous peak representing the ρ meson. It is hard to avoid including it (and all its $SU(3)$ partners) if we want a realistic description. In fact it has been found[11] that many of the values of the counterterms can be numerically understood just with vector meson pole dominance. These arguments strongly suggest the suitability of a model of type

ii. Should we then give up the CPT program? Here, we would like to argue for a model combining the two approaches.

Immediately, there may be a number of objections. First it seems to be discouraging to have to include every particle multiplet with the same kind of microscopic detail that has been applied to the pseudoscalar multiplet. In response, we may note that a natural continuation of the present CPT program would be simply to include at first just the vector meson multiplet. This provides a "clean break" in the sense of retaining just the low lying S-wave quark anti-quark states in the model. It would provide coverage of the region up to around 1 GeV. The lessons learned in such a generalization may show us how to economically include the still more massive states.

Another possible objection is related to the general feeling that, since the low lying pseudoscalars are approximate Nambu-Goldstone bosons, they should be treated differently from the other multiplets. This apparent objection would appear to be strengthened by the folk wisdom that while one can explain the properties of a "normal" multiplet like the vectors with a minimal deviation from $SU(3)$ symmetry, much more elaboration, via the inclusion of many arcane symmetry breaking terms on the CPT list of counterterms, is required for the pseudoscalar multiplet. Here, we would like to point out that this folk wisdom does not seem to hold. In a recent paper,[12] which should be consulted for more explanations and references, it is shown that exactly analogous symmetry breaking terms are required for both the vector and pseudoscalar multiplets at the Okubo-Zweig-Iizuka (OZI) rule conserving level. This suggests that all multiplets be treated in the same way. The (approximate) spontaneous breakdown of chiral symmetry is certainly a crucial feature but it would appear to affect *every* multiplet, presumably via the "pion cloud" intrinsic to each particle. Of course, the vector and higher multiplets have non-zero masses in the chiral limit. One might imagine that the appropriate "large" scale with which to compare the effects of the perturbation $(\hat{m}\mathcal{M})$ is $(\alpha')^{-1/2} \simeq 1.06$ GeV, α' being the universal Regge slope parameter. The best choice of renormalization point for the loop diagrams requires investigation.

In Ref. 12 the vector meson nonet field $\rho_\mu(x)$ is introduced, for convenience, in terms of auxiliary, linearly transforming "gauge fields" A_μ^L and A_μ^R by[13]:

$$A_\mu^L = \xi\rho_\mu\xi^\dagger + \frac{i}{\tilde{g}}\xi\partial_\mu\xi^\dagger, \; A_\mu^R = \xi^\dagger\rho_\mu\xi + \frac{i}{\tilde{g}}\xi^\dagger\partial_\mu\xi, \quad\quad (2.5)$$

where $\xi = U^{1/2}$ and $\tilde{g}$ is related to the $\rho\phi\phi$ coupling constant. ρ_μ transforms non-linearly in this description, which corresponds to eliminating the axial vector mesons in analogy to the elimination of the scalar mesons which led to the non linearly transforming pseudoscalar multiplet inside U. Note that both the scalars and axials are P-wave $q\overline{q}$ bound states so this truncation is conceptually consistent.

Now we can play the CPT game, constructing all chiral invariants and covariants up to a certain order in derivatives. Both vector and pseudoscalar fields would be included. This should necessitate readjusting the coefficients of those terms

containing only pseudoscalars which were dominated by vector meson exchange. To start to explore this rather complicated scheme we will, first of all, specialize to the symmetry breaking terms and try to fit all the mass differences, including those which are isospin violating. We will also fit the meson decay constants and $V \to \phi\phi$ decay widths. As a physical approximation we shall demand that (with one significant exception) the symmetry breaking terms be single traces in flavor space and that all field matrices represent nonets. This is Okubo's form[14] of the OZI rule and appears to be respected by the existing CPT fit for the pseudoscalar only symmetry breakers. The final approximation is to neglect the "chiral logs". This also works in the existing CPT fits. Of course, these approximations can be relaxed in the future.

Then the symmetry breaking terms which conserve the OZI rule are taken to be (up to quartic order in derivatives):

$$\mathcal{L}_{SB} = \text{Tr} \{\mathcal{M}[\delta'(U + U^\dagger - 2) + \alpha'(A^L_\mu U A^R_\mu + A^R_\mu U^\dagger A^L_\mu)$$

$$+ \beta'(\partial_\mu U \partial_\mu U^\dagger U + U^\dagger \partial_\mu U \partial_\mu U^\dagger) + \gamma'(F^L_{\mu\nu} U F^R_{\mu\nu} + F^R_{\mu\nu} U^\dagger F^L_{\mu\nu})]$$

$$+ \lambda'^2[\mathcal{M}U^\dagger \mathcal{M}U^\dagger + \mathcal{M}U\mathcal{M}U - 2\mathcal{M}^2] + \mu'(A^L_\mu \mathcal{M} A^R_\mu \mathcal{M})\} \qquad (2.6)$$

where $F^{L,R}_{\mu\nu} = \partial_\mu A^{L,R}_\nu - \partial_\nu A^{L,R}_\mu - i\tilde{g}[A^{L,R}_\mu, \ A^{L,R}_\nu]$ while $\alpha', \beta', \gamma', \delta', \lambda'^2, \mu'$ are constants to be determined. Notice that there are three analogous vector terms and pseudoscalar terms. Physically, each multiplet has a non-derivative and a derivative type symmetry breaker proportional to $\mathcal{M}$ as well as a non-derivative term proportional to $\mathcal{M}^2$. It turns out that they are *all* required to fit the pseudoscalar and vector particle properties mentioned above.

As explained in section III of Ref. 12 it is convenient to determine a suitable number of the physical quantities while holding the quark mass ratio x constant. Then there are three predictictions for each value of x. A best fit is obtained for the quark mass ratios:

$$x = 37 \ , \quad y = -0.36. \qquad (2.7)$$

If the γ' term for the vectors were not present it would be very difficult to get reasonable predictions for the non-electromagnetic part of $m(K^{*0}) - m(K^{*+})$ and for the width ratio $\Gamma(K^*)/\Gamma(\rho)$. The effect of this derivative type symmetry breaking term is to introduce non-trivial wave function renormalizations for the K^* and ϕ particles. Similarly, the μ' term improves the predictions of the mass and width of the ϕ meson in the present framework.

For the pseudoscalars, it is well known that we can not restrict ourselves to just the OZI rule conserving terms. The needed extra terms are discussed in sections II(c) and IV of Ref. 12. There it is shown that the minimal Lagrangian which can solve the $U(1)$ problem (with the aid of an auxiliary glueball field) can be modified by the addition of suitable symmetry breaking terms to give a reasonable description of the $\eta - \eta'$ system. In particular, the old problem[15] of too small η mass, which

has more recently attracted some attention,[16] was solved. A consequence of this discussion is the presence of the OZI rule violating term $\propto$

$$\{\text{Tr } [\mathcal{M}(\text{U} - \text{U}^\dagger)]\}^2. \qquad (2.8)$$

On the other hand, there is no special reason to include a sizeable term of the type

$$\{\text{Tr } [\mathcal{M}(\text{U} + \text{U}^\dagger)]\}^2. \qquad (2.9)$$

The inclusion of (2.8) but not (2.9), which is very natural in the approach mentioned above, amounts to a practical resolution of the Kaplan-Manohar ambiguity.[17] Some discussion is given in section VII(c) of Ref. 12. We also have no special reason to include the pseudoscalar OZI rule violating term of the type $\propto$

$$\text{Tr } (\partial_\mu \text{U} \partial_\mu \text{U}^\dagger) \text{ Tr } [\mathcal{M}(\text{U} + \text{U}^\dagger)]. \qquad (2.10)$$

This term is found to be very small in the usual CPT fit with pseudoscalars only. We have now accounted for all the terms depending upon $\mathcal{M}$ on the CPT list for pseudoscalars. The only important OZI rule violating one was, as expected, the one associated with the $U(1)$ problem and the $\eta - \eta'$ system. Thus our identification of the most important symmetry breaking terms seems justified for our purpose of making an appraisal of the validity of this model.

Incidentally, we remark that the quark mass ratios in (2.7) are somewhat different from the usual[18] ones $x = 25.0 \pm 2.5$, $y = -0.28 \pm 0.03$. With our determination and a *choice* $m_s(1 \text{ GeV})$=0.175 GeV we would have m_u=3.2 MeV and m_d=7.9 MeV.

To sum up, we have presented evidence that, apart from the special terms needed to solve the $U(1)$ problem, the treatment of symmetry breaking in the pseudoscalar and vector multiplets involves exactly analogous terms. The implication is that chiral perturbation theory might be extensible, as outlined, to the vectors too in order to model low energy QCD up to around 1 GeV. Certainly a large number of processes, including loop contributions, must be examined to fully test this idea. But the first step is to make a preliminary calculation by picking out the terms expected to be most important and working at tree level. The treatment of symmetry breaking discussed here in that manner provides an optimistic sign.

3. Acknowledgments

4. References

1. E.C.G. Sudarshan and R.E. Marshak, *Proceedings of the Padua conference on mesons and recently discovered particles*, p. V-14 (1957). See also R.P. Feynman and M. Gell-Mann, *Phys. Rev.* **109**, 193 (1958) and J.J. Sakurai, *Nuovo Cimento* **7**, 649 (1958).

2. R.E. Marshak, N. Mukunda and S. Okubo, *Phys. Rev* **137** B698 (1965).

3. Y. Nambu in *Group Theoretical Concepts and Methods in Elementary Particle Physics*, edited by F. Gürsey (Gordon and Breach, New York, 1964).

4. K. Nishijima, *Nuovo Cimento*, **11** 698 (1959); F. Gürsey, *Ann. Phys.* (N.Y.) **12** 91 (1961); J. Cronin, *Phys. Rev.* **161**, 1483 (1967).

5. R.E. Marshak, Riazuddin and C.P. Ryan, *Theory of Weak Interactions in Particle Physics*, Wiley-Interscience (1969).

6. M. Gell-Mann, *Phys. Rev.* **125**, 1067 (1962).

7. L.-F. Li and H. Pagels, *Phys. Rev. Lett.* **26**, 1204 (1971); S. Weinberg, *Physica* **96A**, 327 (1979); J. Gasser and L. Leutwyler, *Nucl. Phys.* **250**, 465 (1985).

8. G. Veneziano, *Nuovo Cimento* **57A**, 190 (1968); See also L. Van Hove, *Phys. Lett.* **24B**, 183 (1967).

9. G. 't Hooft, *Nucl. Phys.* **B72**, 461 (1974); E. Witten *ibid*, **B160**, 57 (1979).

10. T. Skyrme, *Proc. Roy. Soc.* **A262**, 237 (1961).

11. J. Donoghue, C. Ramirez and G. Valencia, *Phys. Rev.* **D39**, 1947 (1989); G. Ecker, J. Gasser, A.Pich and E. de Rafael, *Nucl. Phys.* **B221**, 311 (1989).

12. J. Schechter, A. Subbaraman and H. Weigel, *Phys. Rev.* **D48**, 339 (1993).

13. Ö. Kaymakcalan and J. Schechter, *Phys. Rev.* **D31**, 1109 (1985). An alternate equivalent approach is given by T. Fujiwara, T. Kugo, H. Terao, S. Uehara and K. Yamawaki, *Prog. Theor. Phys.* **73**, 926 (1985). The equivalence is demonstrated in P. Jain, R. Johnson, U. Meissner, N.W. Park and J. Schechter, *Phys. Rev.* **D37**, 3252 (1988).

14. S. Okubo, *Phys. Lett.* **5**, 165 (1963).

15. See Fig. 2 of V. Mirelli and J. Schechter, *Phys. Rev.* **D15** 1361 (1977).

16. H. Georgi, *Phys. Rev.* **D49**, 1666 (1994); S. Peris , CERN preprint TH 7109/93.

17. D. Kaplan and A. Manohar, *Phys. Rev. Lett.* **56**, 2004 (1986).

18. J. Gasser and H. Leutwyler, *Phys. Rep.* **87**, 77 (1982).